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On the integration of a certain matrix element $\langle 0|f^{(2)}(x)|q, k\rangle$ in the perturbation approach of quantum electrodynamics

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§ 1. The matrix element $\langle 0|f^{(2)}(x)|q, k\rangle$

The coupled equations of motion of a Dirac field and an electromagnetic field in interaction are

$$\left(\gamma\frac{\partial}{\partial x}+m\right)\psi(x)=ie\gamma A(x)\psi(x)\equiv f(x), \tag{1}$$

$$\Box A_\mu(x)=-\frac{ie}{2}[\bar\psi(x),\gamma_\mu\psi(x)]\equiv -j_\mu(x). \tag{2}$$

For the coupled fields one is not able to obtain exact solutions. If one makes a perturbation approach one finds that the high order approximations of the field operators become extremely complicated. The second order approximation of the operator $f(x)$ in the Heisenberg representation can be written

$$\begin{aligned} f^{(2)}(x)=-\frac{i}{8}\int\int dx'\,dx''\,\{&\gamma_\lambda S_R(x-x')\gamma_{\lambda'}S_R(x'-x'')\gamma_{\lambda''}\psi^{(0)}(x'')\times\\ &\times\{A_\lambda^{(0)}(x)\{A_{\lambda'}^{(0)}(x'),A_{\lambda''}^{(0)}(x'')\}\}-\gamma_\lambda S_R(x-x')\gamma_{\lambda'}\{\psi^{(0)}(x'),[\bar\psi^{(0)}(x''),\gamma_{\lambda''}\psi^{(0)}(x'')]\}\times\\ &\times[A_\lambda^{(0)}(x)\delta_{\lambda'\lambda''}D_R(x'-x'')+A_{\lambda'}^{(0)}(x')\delta_{\lambda\lambda''}D_R(x-x'')]-\\ &-\gamma_\lambda\{\psi^{(0)}(x),[\bar\psi^{(0)}(x')\gamma_{\lambda'}\underset{R②}{S_{A①}}(x'-x''),\gamma_{\lambda''}\psi^{(0)}(x'')]\}\times\\ &\times[A_{\lambda'}^{(0)}(x')\underset{①}{\delta_{\lambda\lambda''}}D_R(x-x'')+A_{\lambda''}^{(0)}(x'')\underset{②}{\delta_{\lambda\lambda'}}D_R(x-x')]\}. \end{aligned} \tag{3}$$

Here $S_R(x)$, $S_A(x)$ and $D_R(x)$ have the following integral representations

$$\underset{A}{S_R}(x)=\frac{1}{(2\pi)^4}\int dp\,e^{ipx}(i\gamma p-m)\left[P\frac{1}{p^2+m^2}\pm i\pi\varepsilon(p)\,\delta(p^2+m^2)\right] \tag{4}$$

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and
$$D_R(x)=\frac{1}{(2\pi)^4}\int d\,k\,e^{ikx}\left[P\frac{1}{k^2}+i\pi\,\delta(k^2)\,\varepsilon(k)\right],\tag{5}$$

where
$$\varepsilon(k)=\frac{k_0}{|k|}.$$

It is the purpose of the present paper to demonstrate how one can perform the integrations necessary for evaluating the matrix element $\langle 0\,|\,f^{(2)}(x)\,|\,q,k\rangle$, where $|\,q,k\rangle$ represents a state with one electron and one photon whereas $\langle 0\,|$ represents vacuum. Only the aspects of the calculation which present some difficulty are included here.

For the free electromagnetic field we want to use the following relations

$$\langle 0\,|\,A_\nu^{(0)}(x)\,|\,k\rangle=\frac{e_\nu^{(\lambda)}}{\sqrt{2\,V\,\omega}}\,e^{ikx},\tag{6}$$

$$\langle k\,|\,A_\nu^{(0)}(x)\,|\,0\rangle=\frac{e_\nu^{(\lambda)}}{\sqrt{2\,V\,\omega}}\,e^{-ikx},\tag{7}$$

where $e_\nu^{(\lambda)}$ are the polarization vectors, for which

$$\sum e_\nu^{(\lambda)}\,e_\mu^{(\lambda)}=\delta_{\mu\nu},\tag{8}$$

and for the free Dirac field

$$\langle 0\,|\,\psi^{(0)}(x)\,|\,q\rangle=\frac{1}{\sqrt{V}}\,u_r^{(+)}(q)\,e^{iqx}\tag{9}$$

$$\langle 0\,|\,\bar\psi^{(0)}(x)\,|\,q'\rangle=\frac{1}{\sqrt{V}}\,\bar u_{r'}^{(-)}(-q')\,e^{iq'x},\tag{10}$$

$$\langle q'\,|\,\psi^{(0)}(x)\,|\,0\rangle=\frac{1}{\sqrt{V}}\,u_{r'}^{(-)}(-q')\,e^{-iq'x},\tag{11}$$

$$\langle q\,|\,\bar\psi^{(0)}(x)\,|\,0\rangle=\frac{1}{\sqrt{V}}\,\bar u_r^{(+)}(q)\,e^{-iqx},\tag{12}$$

with
$$\sum_{\text{spins}} u_\alpha^{(+)}(q)\,\bar u_\beta^{(+)}(q)=\frac{-1}{2\,E}\,(i\gamma q-m)_{\alpha\beta},\tag{13}$$

$$\sum_{\text{spins}} u_\alpha^{(-)}(-q')\,\bar u_\beta^{(-)}(-q')=\frac{-1}{2\,E'}\,(i\gamma q'+m)_{\alpha\beta}.\tag{14}$$

The notation V stands for the volume of periodicity. When $V\to\infty$ we have formally

$$\frac{1}{V}\sum_{\boldsymbol p} f(\boldsymbol p)\to\frac{1}{(2\pi)^3}\int d^3p\,f(\boldsymbol p).\tag{15}$$

If we use the relations (3)–(15) to calculate $\langle 0 | f^{(2)}(x) | q, k\rangle$ we obtain

$$\begin{aligned}\langle 0 | f^{(2)}(x) | q, k\rangle = & \frac{-i}{(2\pi)^3}\cdot\frac{1}{2}\int d k' \Big\{\gamma_\nu \frac{i\gamma q - m}{[q^2+m^2]_R}\gamma_\lambda [i\gamma(q-k')-m]\gamma_\lambda \times \\ & \times \left[\frac{\delta(k'^2+\mu^2)}{[(q-k')^2+m^2]_R} + \frac{\delta((q-k')^2+m^2)}{[k'^2+\mu^2]_R}\right] + \gamma_\lambda[i\gamma(q+k-k')-m]\gamma_\nu[i\gamma(q-k')-m]\gamma_\lambda \times \\ & \times \left[\frac{\delta(k'^2+\mu^2)}{[(q+k-k')^2+m^2]_R[(q-k')^2+m^2]_R} + \frac{\delta((q-k')^2+m^2)}{[(q+k-k')^2+m^2]_R[k'^2+\mu^2]_R} + \right. \\ & \left. + \frac{\delta((q+k-k')^2+m^2)}{[(q-k')^2+m^2]_A[k'^2+\mu^2]_R}\right] + \gamma_\lambda[i\gamma(q+k-k')-m]\gamma_\lambda \frac{[i\gamma(q+k)-m]}{[(q+k)^2+m^2]_R}\gamma_\nu \times \\ & \times \left[\frac{\delta(k'^2+\mu^2)}{[(q+k-k')^2+m^2]_R} + \frac{\delta[(q+k-k')^2+m^2]}{[k'^2+\mu^2]_R}\right]\Big\} \langle 0 | \psi^{(0)}(x) | q\rangle \langle 0 | A_\nu^{(0)}(x) | k\rangle - \\ & - \frac{i}{(2\pi)^3}\cdot\frac{1}{2}\int \frac{d k'}{(k'^2+\mu^2)_R}\gamma_\lambda \langle 0 | \psi^{(0)}(x) | q\rangle S p \Big\{\Big[\gamma_\nu \frac{i\gamma(k+k')-m}{(k+k')^2+m^2}\gamma_\lambda + \\ & + \gamma_\lambda \frac{i\gamma(k+k')-m}{(k+k')^2+m^2}\gamma_\nu\Big](i\gamma k'+m)\Big\}\delta(k'^2+m^2)\langle 0 | A_\nu^{(0)}(x) | k\rangle. \end{aligned} \tag{16}$$

In performing the integrations in (16) one finds that the contribution that requires a certain amount of effort is the following

$$\begin{aligned} K_\mu(p, p') = -\frac{1}{(2\pi)^3}\cdot\frac{1}{2}\int d k\, \gamma_\lambda(i\gamma(p-k)-m)\gamma_\mu(i\gamma(p'-k)-m)\gamma_\lambda \times \\ \times P\frac{\delta(k^2+\mu^2)\,\delta((p-k)^2+m^2)}{(p'-k)^2+m^2}[1-\varepsilon(k)\,\varepsilon(k-p)], \end{aligned} \tag{17}$$

where $p = q+k$, $p' = q$ and where we have dropped the prime on the k in the integrand.

§ 2. Method of integration

The evaluation of $K_\mu(p, p')$ can be reduced to the following three types of integrals

$$I_0 = \int d k\, \delta(k^2+\mu^2)\,\delta((p-k)^2+m^2)\,\delta((p'-k)^2+\varkappa)[1-\varepsilon(k)\,\varepsilon(k-p)], \tag{18}$$

$$I_\mu = \int d k\, k_\mu\, \delta(k^2+\mu^2)\,\delta((p-k)^2+m^2)\,\delta((p'-k)^2+\varkappa)[1-\varepsilon(k)\,\varepsilon(k-p)], \tag{19}$$

$$I_{\mu\nu} = \int d k\, k_\mu k_\nu\, \delta(k^2+\mu^2)\,\delta((p-k)^2+m^2)\,\delta((p'-k)^2+\varkappa)[1-\varepsilon(k)\,\varepsilon(k-p)], \tag{20}$$

where the integration over $\varkappa$ remains to be done.

The first of these three integrals (18) is simple to calculate and yields

$$I_0 = \frac{\pi}{2\sqrt{\lambda}}\,\theta(\phi(\varkappa))\,\theta(-p^2 - m^2 + \mu^2), \tag{21}$$

where

$$\lambda = (pp')^2 - p^2 p'^2 \tag{22}$$

and

$$\phi(\varkappa) = -4\lambda\mu^2 + p'^2(p^2 + m^2 - \mu^2)^2 + p^2(p'^2 + \varkappa - \mu^2)^2 - 2pp'(p^2 + m^2 - \mu^2)(p'^2 + \varkappa - \mu^2). \tag{23}$$

We have here introduced the convenient notation

$$\theta(x) = \tfrac{1}{2}[1 + \varepsilon(x)].$$

The remaining two integrals can be treated in the following way. In the integrand of (19) we substitute $k_\mu \sim \alpha p_\mu + \beta p'_\mu$. This we are allowed to do for invariance reasons. If we then multiply (19) by p_μ and p'_μ we obtain the equations

$$\alpha p^2 + \beta pp' = \tfrac{1}{2}(p^2 + m^2 - \mu^2), \tag{24}$$

$$\alpha pp' + \beta p'^2 = \tfrac{1}{2}(p'^2 + \varkappa - \mu^2), \tag{25}$$

respectively, where we have used the following relations

$$\begin{aligned} pk &= \tfrac{1}{2}(p^2 + m^2 - \mu^2), \\ p'k &= \tfrac{1}{2}(p'^2 + \varkappa - \mu^2), \end{aligned} \tag{26}$$

which follow directly from the δ-functions. From (19), (24) and (25) we then have the result

$$I_\mu = (\alpha p_\mu + \beta p'_\mu)\,I_0, \tag{27}$$

where

$$\alpha = \frac{1}{2\lambda}[pp'(p'^2 + \varkappa - \mu^2) - p'^2(p^2 + m^2 - \mu^2)], \tag{28}$$

$$\beta = \frac{1}{2\lambda}[pp'(p^2 + m^2 - \mu^2) - p^2(p'^2 + \varkappa - \mu^2)]. \tag{29}$$

For $I_{\mu\nu}$ we write in a similar way

$$I_{\mu\nu} = [(\alpha p_\mu + \beta p'_\mu)(\alpha p_\nu + \beta p'_\nu) + A\,\delta_{\mu\nu} + B\,p_\mu p_\nu + C\,p'_\mu p'_\nu + D\,(p_\mu p'_\nu + p'_\mu p_\nu)]\,I_0. \tag{30}$$

In (30) the product $(\alpha p_\mu + \beta p'_\mu)(\alpha p_\nu + \beta p'_\nu)$ contains terms which could also have been included in B, C and D. The way of writing Eq. (30) which we here use is justified by the formal simplicity thereby obtained in our following procedure. Thus the expressions for A, B, C and D will be simple. To determine these expressions let us multiply Eq. (30) successively by $p_\mu p_\nu$, $p'_\mu p'_\nu$ and $p'_\mu p_\nu$. In doing so we find in each case that the left hand side of Eq. (30) cancels the first product of factors containing α and β on the right hand side. We then obtain the following three relations

$$p^2 X + C\lambda = 0, \tag{31}$$

$$p'^2 X + B\lambda = 0, \tag{32}$$

$$p p' X - D\lambda = 0, \tag{33}$$

where
$$X = A + B p^2 + C p'^2 + D \cdot 2 p p'. \tag{34}$$

To find the remaining relation for A, B, C and D to be determined we put $\mu = \nu$ in (20) and (30). This yields

$$\alpha^2 p^2 + \beta^2 p'^2 + 2\alpha\beta p p' + 3A + X = -\mu^2. \tag{35}$$

The solution will be

$$\begin{aligned} &X = -\frac{1}{8\lambda}\phi, \\ &A = \frac{1}{8\lambda}\phi, \quad B = \frac{p'^2}{8\lambda^2}\phi, \\ &C = \frac{p^2}{8\lambda^2}\phi, \quad D = -\frac{p p'}{8\lambda^2}\phi. \end{aligned} \tag{36}$$

§ 3. Evaluation of the integral $K_\mu(p, p')$

After these preliminaries we can write

$$K_\mu(p, p') = \frac{1}{(2\pi)^3} \cdot \frac{1}{2} \cdot \frac{\pi}{2\sqrt{\lambda}} \theta(-p^2 - m^2 + \mu^2) \int_{-\infty}^{\infty} \frac{d\varkappa}{\varkappa - m^2} \Gamma_\mu(p, p', \varkappa)\, \theta(\phi(\varkappa)), \tag{37}$$

where
$$\Gamma_\mu = (i\gamma p + m)(f_1 \gamma_\mu + f_2 i p_\mu + f_3 i p'_\mu) + g_1 \gamma_\mu + g_2 i p_\mu + g_3 i p'_\mu. \tag{37 a}$$

We are only interested in $K_\mu(p, p')$ when $p'^2 = q^2 = -m^2$, as follows from one of the δ-functions in Eq. (17). Terms with a factor $(i\gamma p' + m)$ to the right give no contribution because this factor operates on a free field operator in the final integral. Utilizing the property of the γ matrices

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu} \tag{38}$$

and the well known identities

$$\gamma_\lambda \gamma_\mu \gamma_\lambda = -2\gamma_\mu, \tag{39}$$

$$\gamma_\lambda \gamma_\mu \gamma_\nu \gamma_\lambda = 4\delta_{\mu\nu} \tag{40}$$

we can expand $\gamma_\lambda (i\gamma(p-k) - m)\gamma_\mu (i\gamma(p'-k) - m)\gamma_\lambda$ in (17).

Making use of Eqs. (27) and (30) and observing the relations

$$\begin{aligned} &i\gamma p\gamma_\mu i\gamma p \sim 2ip'_\mu i\gamma p + p^2\gamma_\mu,\\ &i\gamma p'\gamma_\mu i\gamma p' \sim 2imp'_\mu + p'^2\gamma_\mu,\\ &i\gamma p'\gamma_\mu i\gamma p \sim 2ip'_\mu i\gamma p + 2\gamma_\mu pp' - 2imp_\mu + mi\gamma p\gamma_\mu, \end{aligned} \tag{41}$$

we then obtain in (37 a)

$$\begin{aligned} f_1 &= 2m(\beta+\alpha-1),\\ f_2 &= 4\left(\alpha-\alpha^2-p'^2\frac{\phi}{8\lambda^2}\right),\\ f_3 &= 4\left(\alpha+\beta-\alpha\beta-1+pp'\frac{\phi}{8\lambda^2}\right),\\ g_1 &= 2p^2\alpha(1-\alpha)+2p'^2\beta(1-\beta)+4pp'(\alpha+\beta-\alpha\beta-1)-2m^2(\beta+\alpha),\\ g_2 &= 4m(-\beta+\alpha^2+\alpha\beta+(p'^2-pp'))\frac{\phi}{8\lambda^2},\\ g_3 &= 4m\left(-\alpha+\beta^2+\alpha\beta+(p^2-pp')\frac{\phi}{8\lambda^2}\right). \end{aligned} \tag{42}$$

In a similar way let us write

$$K_\mu(p,p') = (i\gamma p+m)(G_1\gamma_\mu+G_2ip_\mu+G_3ip'_\mu)+H_1\gamma_\mu+H_2ip_\mu+H_3ip'_\mu. \tag{43}$$

We now perform the $\varkappa$ integration in (37), using the relations (28), (29) and (42) and remembering that $p'^2=-m^2$ and $(p'-p)^2=-\mu^2$. This yields

$$G_1 = -\frac{1}{8\pi^2}\cdot\frac{m}{p^2}, \tag{44}$$

$$G_2 = \frac{1}{8\pi^2}\cdot\frac{1}{p^2+m^2}\left\{\frac{1}{p^4}(p^2-m^2)(p^4-6p^2m^2-m^4)+\frac{4m^2}{p^2+m^2}(p^2-2m^2)\log\left(-\frac{p^2}{m^2}\right)\right\}, \tag{45}$$

$$G_3 = -\frac{3}{4\pi^2}\cdot\frac{p^2-m^2}{(p^2+m^2)^2}\left\{\frac{m^2}{p^2+m^2}\log\left(-\frac{p^2}{m^2}\right)+\frac{p^2-m^2}{2p^2}\right\}, \tag{46}$$

$$\begin{aligned} H_1 = -\frac{1}{64\pi^2}\cdot\frac{1}{\lambda^{\frac{3}{2}}}\Bigg\{&\frac{2\lambda}{p^2}[(p^2+m^2)(5m^2-p^2)-8m^2\mu^2+3\mu^4]-\\ &-[m^2(p^2+m^2)^2+\mu^2(p^4+5m^4-2m^2p^2-10m^2\mu^2+4p^2\mu^2+4\mu^4)]\times\\ &\times\log\frac{p^2(p^2+m^2)+2p^2\mu^2}{-m^2(p^2+m^2)+2m^2\mu^2-\mu^4}\Bigg\}, \end{aligned} \tag{47}$$

where $$4\lambda = (p^2 + m^2 + \mu^2)^2 - 4m^2\mu^2,$$

$$H_2 = \frac{m}{8\pi^2}\left\{\frac{(p^2 - m^2)(m^2 + 6p^2)}{p^4(p^2 + m^2)} - 2\cdot\frac{p^2 - 4m^2}{(p^2 + m^2)^2}\log\left(-\frac{p^2}{m^2}\right)\right\} \tag{48}$$

$$H_3 = -\frac{m}{8\pi^2}\left.\frac{7p^2 - 3m^2}{p^2(p^2 + m^2)} - 2\frac{p^2 - 4m^2}{(p^2 + m^2)^2}\log\left(-\frac{p^2}{m^2}\right)\right\}. \tag{49}$$

In calculating the Hilbert transform[1]

$$\bar{H}_1(m^2) = \int\limits_{(m+\mu)^2}^{N}\frac{da\,H_1(-a)}{a - m^2} \qquad (a = -p^2) \tag{50}$$

we separate $\bar{H}_1(m^2)$ into two parts

$$\bar{H}_1(m^2) = (\bar{H}_1(m^2))_{\mathrm{I}} + (\bar{H}_1(m^2))_{\mathrm{II}}, \tag{51}$$

where the second term contains the logarithm. Thus

$$(\bar{H}_1(m))_{\mathrm{I}} = \frac{1}{16\pi^2}\int\limits_{(m+\mu)^2}^{N}\frac{da}{a(a - m^2)}\cdot\frac{(m^2 - a)(5m^2 + a) - 8m^2\mu^2 + 3\mu^4}{\sqrt{(m^2 - a + \mu^2)^2 - 4m^2\mu^2}}, \tag{52}$$

$$(\bar{H}_1(m^2)_{\mathrm{II}} =$$

$$= \frac{1}{8\pi^2}\int\limits_{(m+\mu)^2}^{N}\frac{da}{a - m^2}\cdot\frac{m^2(m^2 - a)^2 + \mu^2(a^2 + 2am^2 + 5m^4 - 10m^2\mu^2 - 4a\mu^2 + 4\mu^4)}{[(m^2 - a + \mu^2)^2 - 4m^2\mu^2]^{\frac{3}{2}}}\times$$

$$\times\log\frac{a(a - m^2) - 2a\mu^2}{m^2(a - m^2) + 2m^2\mu^2 - \mu^4}. \tag{53}$$

Straightforward calculation gives

$$(\bar{H}_1(m^2))_{\mathrm{I}} = \frac{1}{16\pi^2}\left(5\log\frac{\mu}{m} - \log N + \log m\mu\right). \tag{54}$$

In order to calculate $(\bar{H}_1(m^2))_{\mathrm{II}}$ we devide the interval of integration into two parts and let N tend to infinity

$$(\bar{H}_1(m^2))_{\mathrm{II}} = \int\limits_{(m+\mu)^2}^{m^2(1+\delta)} + \int\limits_{m^2(1+\delta)}^{\infty} = (\bar{H}_1(m^2))_{\mathrm{II\,a}} + (\bar{H}_1(m^2))_{\mathrm{II\,b}}, \tag{55}$$

where $$\delta \gg \frac{2\mu}{m}.$$

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[1] G. Källén: Quantenelektrodynamik, Handbuch der Physik, Band V, Teil 1.

In $(\bar{H}_1(m^2))_{\text{II a}}$ we substitute

$$2m\mu t = a - m^2 - \mu^2 + \sqrt{(a - m^2 - \mu^2)^2 - 4m^2\mu^2} \tag{56}$$

to obtain

$$(\bar{H}_1(m^2))_{\text{II a}} = \frac{1}{8\pi^2}\int_1^{\delta\frac{m}{\mu}} dt\,\frac{t}{(t^2+1)^2}\left[\left(\frac{t^2+1}{t}\right)^2 + 8\right] =$$

$$= \frac{1}{8\pi^2}\left[\log\delta\frac{m}{\mu} + 2 - \frac{1}{\delta^2\left(\frac{m}{\mu}\right)^2 + 1}\right] \xrightarrow[\delta\frac{m}{\mu} \gg 2]{} \frac{1}{8\pi^2}\left(\log\delta + \log\frac{m}{\mu} + 2\right), \tag{57}$$

$(\bar{H}_1(m^2))_{\text{II b}}$ we integrate by part. This yields

$$(\bar{H}_1(m^2))_{\text{II b}} = \frac{m^2}{8\pi^2}\left(\left.\int_{m^2(1+\delta)}^{\infty} - \frac{1}{a+m^2}\log\frac{a}{m^2}\right. + \int_{m^2(1+\delta)}^{\infty} da\,\frac{1}{a-m^2}\cdot\frac{1}{a}\right) = \frac{1}{8\pi^2}\left(1 + \log\frac{1}{\delta}\right). \tag{58}$$

Summing up we obtain

$$(\bar{H}_1(m^2))_{\text{II}} = \frac{1}{8\pi^2}\left(3 + \log\frac{m}{\mu}\right) \tag{59}$$

and finally from Eqs. (51), (54) and (59)

$$\bar{H}_1(m^2) = -\frac{1}{16\pi^2}\left(4\log\frac{m}{\mu} + \log\frac{N}{m^2} - 6\right). \tag{60}$$

The remaining Hilbert transforms are straightforward.

The functions we have determined enter in calculating observable quantities, e.g. the magnetic moment of the electron and also in studies on the Compton scattering.

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